Models for harnessing the Internet in mathematics education
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Abstract
In recent years, the Internet has increasingly been used to provide significant resources for student to learn mathematics and to learn about mathematics, as well as significant resources for teachers to support these. Effective access to and use of these has been hampered in practice by limited facilities in schools and the limited experience of many mathematics teachers with the Internet for mathematical purposes. This paper offers models for understanding the effective use of Internet resources, based on typologies of resources for learning and teaching mathematics. Six categories of Internet resources for mathematics student use are identified: (i) Interactive resources; (ii) Reading interesting materials; (iii) Reference information; (iv) Communication; (v) Problem solving; and (vi) Webquests. Similarly, five categories of Internet resources for mathematics teacher use are identified: (i) Lesson preparation; (ii) Official advice and support; (iii) Professional engagement; (iv) Commercial activity and support; and (v) Local school web sites. The paper recognises that web resources can be used in a range of ways, including supporting both teaching and learning. The prospects for sound use of the Internet are briefly described in terms of these models of use.

Introduction
Although the situation is different from one country to the next, the Internet has continued to rise in significance across the developed world, with the mounting prosperity of the last two decades and with the rapidly declining infrastructure costs to individuals and to schools of having reasonable access. In this paper, we consider some of the ways in which the Internet has been harnessed to date for use in mathematics education, in order to construct a model of a productive relationship.

The Internet has the potential to support the learning of students and the teaching of teachers. In short, the problem addressed by this paper concerns the productive use of the Internet for mathematics education: how can students and teachers best exploit the potential of the Internet in their respective roles in learning and teaching?

Description of the model
The Internet has provided previously undreamt of possibilities for connections within mathematics education, both for students and for their teachers, now part of a globalised world. It is appropriate to consider the twin activities of teaching and learning separately for the purposes of understanding the significance of the Internet for mathematics education. Accordingly, the model described in this paper begins by separately identifying the kinds of opportunities potentially afforded by the Internet for students and teachers. These are different because the roles of students and teachers are different. The paper then proceeds to consider some of the connections between these affordances as well as some of the constraints on their implementation in practice.

The Internet for students
A recent paper (Kissane, 2009b) described in some detail a typology of ways in which the Internet might be a useful learning resource for students. Six categories of Internet resources were described briefly and examples given from within most categories. The companion website (Kissane, 2009a) provides links to many examples, and is maintained to take advantage of new opportunities that become available. In this paper, it is appropriate to describe each of the categories quite briefly.

The first category involves the use of interactive resources, taking advantage of the possibility of designing web objects that students can manipulate directly, using software platforms such as Java and Flash. Virtual manipulatives are one example of such a resource. Within this category, there is an increasing number of well-designed materials intended for direct student use, across a wide range of student ages and levels of sophistication. Some of these, such as the National Library of Virtual Manipulatives (Utah State University, 2009) in the USA, comprise large collections that are well organised, with adequate online help to make them independently accessible by students.

The second category involves the provision of worthwhile reading materials for students. In many circumstances, students seem to have access to a remarkably thin range of contemporary materials regarding mathematics, in part because such materials are not routinely available in many homes and even many school libraries. (Kissane, 2009c). A range of recent and well-written materials in various forms has been produced over recent years (not always for an audience of students), some of which have become accessible to students through the Internet.

The third category of materials involves reference materials of various kinds, such as dictionaries, encyclopedias and databases of mathematically related materials. Once again, it is rare for either homes or schools to provide good reference information that is mathematically informative for students, in stark contrast to the wealth of such materials now available on the Internet.

A fourth category of materials is concerned with communication among students and others. The Internet provides mathematics students with an opportunity to communicate with other students and, unlike the use of email, does not require students to know each other beforehand. In this context, the emphasis is of course on communications related to mathematics, not personal communications, although communications between people necessarily are personal to an extent. Communications typically are concerned with understanding mathematical concepts or solving mathematical problems. Similarly, the web can be used to connect students with teachers (or virtual teachers), prepared to respond to their questions or provide advice and feedback concerning mathematics.
The use of the Internet for problem solving presents a fifth category of learning opportunity for students. Some web sites offer regular problems and puzzles for students of different levels of sophistication, as well as advice and hints on solving them (and guides for teachers). While few school environments or school curricula are bereft of problem-solving opportunities, the Internet can provide an environment for problem solving that has some distinctly different characteristics than those available through a book of problems, including the opportunity for solutions by other students to be presented and discussed in a virtual world.

Finally, the Internet can be used to provide students with structured explorations of situations of mathematical interest in the form of webquests. These generally are constructed (by teachers) to support a group of students tackling a task that has a mathematical flavour and which uses the web as a source of real data relevant to the students’ context, as well as a stimulus to work together with a team to tackle a contemporary problem that is connected with mathematics.

Although these categories are presented here as if they are mutually exclusive, in practice some Internet opportunities for students involve more than one of these at once, such as a website that engages students in exploratory activity, using a virtual manipulative, motivating work on a suitable mathematical problem. However, it is argued that the categories each offer distinctively different opportunities to support and encourage student learning, as well as fostering interest in mathematics; these opportunities were not readily available prior to the development of the Internet.

In terms of addressing the central problem addressed by this paper, it is suggested that this typology captures the key opportunities for learning afforded by the Internet, and that there is an already large, and increasing, collection of good examples of most of these. While no classification of this kind will allow each potentially useful web site to be located in one category and not another, a model for Internet use ought sensibly recognise that quite different kinds of opportunities are available: that the Internet for learners is a multi-dimensional object.

The Internet for teachers

While many of the Internet sites that are relevant to students are also likely to be of interest to mathematics teachers, the Internet provides further and different opportunities for teachers. In this section, these are briefly identified and a first attempt at a typology presented for discussion. A website (Kissane, 2009d) supports the development of this model with online examples. The order of the various categories proposed is not intended to be interpreted as meaningful.

One category of mathematics teacher use concerns accessing direct lesson materials or ideas. Of course, many web sites might suggest ideas to teachers for lessons and classroom tasks: such is the nature of the craft of teaching. However, some websites offer detailed collections of lesson plans for mathematics teachers, usually written by teachers themselves, sharing their more successful lessons with others; the Australian Maths 300 collection (Curriculum Corporation, 2009) is a good example of a subscription-based provision of this kind. While some lesson collections are too strongly related to local curricula and contexts to be very useful to teachers elsewhere, there are inevitable differences in teaching styles and there are various standards of quality assurance used before posting, such lesson collections can offer helpful and practical advice to teachers. In a similar vein, some sites have focussed recently on the provision and sharing of teaching materials (such as software files) for new technologies such as interactive whiteboards; two good examples include the site maintained by an Australian teacher (Boggs, 2009) and the excellent subscription site maintained by Keele University staff in the UK (Miller, 2009).

A second category of use for teachers concerns official communications regarding the curriculum or the governance of education within their environment. At least within Australia, but also elsewhere in the developed world, the Internet is increasingly being harnessed by authorities to provide both guidance and support for teachers undertaking their mandatory roles. This use of the Internet has a clear advantage over previous mechanisms for doing similar tasks: that materials can be readily updated and corrected, so that the most useful advice for teachers can be made available very quickly. A possibly unexpected advantage of this means of supporting teachers is that the information available in one jurisdiction (such as in one of the states in Australia or Germany, or one of the countries in Asia) can be efficiently and quickly accessed by those who reside elsewhere. Although this has not been the intention of the developments, it has certainly been a useful by-product, permitting good ideas for teachers or good ideas for curriculum developers to be rapidly shared. A disadvantage of this category of information dissemination is that the burden—and the costs—of printing are passed from authorities to schools and individuals, which is sometimes quite problematic. An alternative of course involves reading materials online, although many do not find this satisfactory, for a range of reasons.

A third category of Internet use for teachers concerns professional engagement. Over the past decade, voluntary professional associations of mathematics teachers have increasingly come to rely on their Internet sites to communicate effectively and helpfully among an organisation, and more widely to the entire community. Thus, national associations such as the Australian Association of Mathematics Teachers (AAMT), the National Council of Teachers of Mathematics in the USA, the Association of Teachers of Mathematics in the UK and similar bodies in other countries have developed strong web presences to support a variety of functions that were previously neglected or handled with great difficulty. An example of this is the use of the AAMT website (AAMT, 2009) to provide a mechanism for the development of policy responses to official documents such as, in the case of Australia recently, the developments towards a national curriculum for mathematics. Prior to the accessibility of the Internet, it was too difficult in practice for such organisations to democratically seek advice from a wide membership. While it is still difficult, as professional people are usually very busy, the constraints of access have been lifted so that a genuinely collaborative effort at policy development is possible. Professional websites serve a number of other purposes as well; in the case of the AAMT website, a lively email list regularly allows professionals to connect together to discuss issues of the day in a collegial manner with colleagues from around the country; such professional collaboration was simply not possible a generation ago. As well as (voluntary) professional associations, recent years have seen the development and maintenance of large
web sites intended to support the work of teachers as professionals in a range of ways. Two exceptional examples of this are the Math Forum in the USA (Drexel University, 2009) and the government-sponsored National Council for Excellence in Teaching Mathematics (2009) in the UK, with an interesting MatheMatica for teachers, among other forms of professional support and collaboration.

A fourth category of Internet use for teachers concerns websites that have a commercial element, yet still offer opportunities for teachers that were previously inaccessible. This category does not include sites whose sole purpose is to provide opportunities for teachers to buy goods or services (as these are strictly commercial sites, not educational sites), but rather includes a range of sites that offer significant support of various kinds to a professional client base. One example of this is the HotMaths (2009) site in Australia, which offers significant curriculum materials for students, providing a school purchases a subscription that gives them access. While subscription websites might be seen solely as commercial exercises (and, indeed, in some cases, correctly so), in other cases, the subscriptions raised are used essentially to fund development of suitable innovative learning materials, for which developmental costs are generally very high. An excellent example is the Maths 300 website in Australia (Curriculum Corporation, 2009), which offers detailed advice for teachers regarding a large set of innovative mathematics lessons, together with other professional materials and professional developmental help, provided a school subscription has been purchased. Another example of commercially connected Internet use comes from companies marketing computer software, computer hardware (including interactive whiteboards) or calculators. Several sites offer significant support for teachers using, or planning to use, the technology involved, in the form of demonstration software, activities to use with students, access to technical advice and help, upgrading of equipment or software and information about communities of users and opportunities for local professional development. In a similar vein, many school textbook publishers now offer privileged access to materials developed and stored on the web to support their text materials. While some of this sort of activity is doubtless part of the promotional activity of the publishers, in order to capture or retain a market with the lure of extra materials, it also offers students opportunities to learn that were not previously available, targeted to a particular set of curriculum materials; without doubt, this also helps their teachers as well.

A fifth category of teacher use of the Internet concerns local use within a school or a school district. Increasingly in developed countries with high (and rapidly increasing) Internet access, the possibility of teachers using a school Internet site for teaching purposes has become real, and of significance. With some expertise and local support, teachers can use their own websites, or the school website, in order to post lesson materials for students and to engage students within the community of the classroom. In some cases, still fairly rare, more structured learning management systems (such as WebCT and BlackBoard) are used to focus instruction, although this is more common in universities and colleges than it is in schools, which, by their nature, have students in full-time attendance during childhood and adolescent years.

To return again to the central problem addressed by this paper, it is clear that there are many opportunities for supporting the work of teachers via the Internet. As for the case of students, the proposed categories in this model do not provide mutually exclusive sets of opportunities, as several websites might be fairly described as helping in more than one way. The model also helps clarify the complex craft of teaching, especially teaching in the twenty-first century. As well as engaging in what many outside education regard as the only role of teachers, that of designing and executing worthwhile activities for the pupils in their immediate classroom, teachers are simultaneously engaged in other activities as well. These other things include working within an official regulatory environment, sometimes with high public visibility. They also include working with others as an active professional, collaborating independently with like-minded professionals, as well as keeping up to date with recent publishing and technology changes and directions. Even teaching itself has changed a good deal in the age of the Internet, with teachers expected, routinely so in some cases, to develop new skills to handle construction and maintenance of school websites and operate within the constraints of a learning management system.

**Connecting teaching and learning**

The two models for students and teachers on the Internet are presented here as if they are distinct. In fact, of course, teachers are themselves interested in both models, since a key role of the teacher is to frame the experience of students, and it is now important that teachers have a sense of what the Internet offers to students, and which of those offerings are worthy of attention in a particular context. The relationship is not, of course, reciprocal: students are for the most part disinterested in what the Internet has to offer their teacher.

The connection is even stronger than that, however. The teacher in search of lessons is likely to make use of the same websites that are helpful for students learning. Indeed, it is usually through the teacher becoming aware of the websites, evaluating their potential benefits for a class and using them in some way, that students are offered access to them. This access might take a number of forms, ranging from advice to use a particular website for some purpose, using a website for a classroom activity of some kind, making links to relevant websites on a school web page, including a website in a webquest designed by the teacher or even assigning homework to students based around a particular website (in the circumstance where good home access to the Internet is assured).

The categories of Internet use that relate to students accessing the Internet for information or for reading interesting materials are also of relevance to the teacher. As noted earlier, many of these are in fact designed for a wide audience, which certainly includes professional mathematics teachers. Many mathematics teachers have reported informally and favourably to the author about the excellent materials of interest to them as mathematically educated people (not necessarily in their role of teacher) in these two categories. Indeed, some of the material is written for this audience.

**Description of the model success**

The models offered in this paper are theoretical models, designed to provide a sense of perspective on what the Internet
might offer mathematics education. As such, they have not yet been subject to formal testing or validation, although they reflect a good deal of the author’s experience in using the Internet from the perspectives of both teaching and learning. For some years now, the author has maintained a website (Kissane, 2009b) with annotated links to good examples of Internet sites for school students (and their teachers or prospective teachers). Work with teachers in contexts of professional development and with student teachers suggests that the categories suggested identify distinctly different and important kinds of use of the Internet by students. It is clear, too, that the detailed examples of websites in these categories are recognised by teachers, both experienced and less experienced, as plausibly related to effective Internet use.

**Transfer of the model to different environments**

By its nature, the Internet has a measure of portability across environments, although of course it is difficult for this to occur across linguistic divides. Successful use of the Internet by either students or teachers depends a good deal on local circumstances, including especially the ease with which Internet access is available to students (at school and at home) and to teachers (inside and outside their classrooms). Excellent overviews of the range of ways in which effective use can be handled are provided by Alejandre (2005) and Galindo (2005), both of whom recognise the inevitability of constraints on good practice and offer helpful ways of working around them.

It is clear that effective use of the Internet in mathematics education brings new professional demands for teachers. Although it is clear that mathematics education can be supported, informed, improved and inspired through appropriate use of the Internet, achieving these aspirations includes high expectations for support of teachers, significant resource needs in schools and a need for new paradigms for both teaching and learning to be developed. In the best of circumstances, the prospects for harnessing the Internet to improving mathematics education are already promising, although in practice the circumstances are still far short of optimal in many classrooms in developed and affluent countries.

**References**


