Different procedures in argumentation and conjecturation in primary school: An experience with Chinese students

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Abstract
As it is possible to read from numerous research studies (Spagnolo, 2005; Nisbet, 2001, 2003) the reasoning schemes and the cognitive process specifically referred to the argumentation, conjecturation and demonstration phases are not possible to be defined in a universal way without taking into consideration the cultural context in which people live and study (linguistic aspects, philosophical aspects, beliefs and values etc.). This characteristic is evident in a clear manner analyzing different cultures as the Italian and the Chinese. The article aims to augment, in an experimental way, as some of the cultural differences verified trough the Italian and Chinese cultures (we will refer to the epistemological aspect of the two cultures) tacitly influence the didactical sphere of the discipline, putting in evidence differences in the basic nature of the cognitive process utilized by the students. The didactical experience discussed in this work, conducted in multicultural classes in Palermo, aims to analyze the cognitive behavior of pupils aged 3-10 in resolving pre-algebraic problems. The present work is inserted in a broader research project, conducted within the GRIM of Palermo and essentially dedicated to the analysis of the logical argumentative schemes verified in the Chinese and Italian cultures in the resolution of different task (tips of solution, resolute algorithmic, verifiable errors) structured in diverse mathematical contexts, created ad hoc. As for the research methodology, the activity was conducted according to Brousseau’s theoretical framework. The selected concept that constituted the mathematical specific milieu was choice in relation to the arithmetical thought, the algebraic thought, and so to the first approach to the concept of variable and unknown. The collected data have been analyzed in parallel according to a quantitative and qualitative analysis. Quantitative analysis has been conducted using the software CHIC 3.0 for non parametric statistics.

Introduction
What we analyze when we discuss about of the binomials mathematics-culture, fundamental relation for this work and for a large tradition of research in didactic of mathematics, is not only the presentation of specific techniques trough witch a certain group of people treat the mathematical knowledge; but a critic discussion of possible correlations between the cultural context on witch these people live and the treated mathematical concepts that are elaborated and obtained. This is the approach that we are following, even if in a first approximation, in our work. What we are interesting in is in
fact the analysis of the reasoning schemes adopted by the student and strictly interpreted in relation to particular aspects of their own culture; with particular attention on the logical-linguistic problems.

For what we said before and with the aim to discuss the treated problematic, we think that could be useful to present briefly:

1. the role of the history of mathematics as an instrument of observation and analysis of multicultural learning/teaching situations: the case of the Nine Chapter as canon for the construction of mathematics (1st Cent. B.C.-1st Cent. A.D.);

2. the role of natural language in the development of mathematics in the history of thought. (Spagnolo, 1996, 2002);

3. the role of fuzzy logic (an approach of the linguistic type) as an interpretive instrument for the Chinese students of some problem situations in class correlated to “common sense” (Spagnolo, 2002, 2005). The main references are those of Zadeh, (as regards the fundamental considerations of fuzzy logic of the linguistic approach) and of Kosko (1995) as regards the relationships and analogies identified between fuzzy logic and oriental thinking.

The first of the reference, the historic and historic-epistemological analysis of mathematical thinking, according to us, could be useful to study the different patterns of reasoning (deducing, conjecturing, demonstrating) in the European and Chinese cultures. This kind of analysis is conducted with the typical argumentations of history and epistemology and it is the basic reference for all the work. In this specific case we are discussing to the principal reference for mathematics in Chinese education: the “Nine Chapters on Mathematical Procedures”.

The second and the third reference specifically refer to the initial acquisition of “symbols” and variable (in a pre-algebraic sense) in children and so an interpretation of the reasoning scheme referred to this.

The “Nine Chapters on Mathematical Art (Jiuzhang suanshu)”, typical Chinese reasoning schemes and possible East Asian Identity in Mathematics Education.

The Jiuzhang suanshu or Nine Chapters on the Mathematical Art is a practical handbook of mathematics consisting of 246 everyday problems of engineering, surveying, trade, and taxation. It played a fundamental role in the development of mathematics in China, not dissimilar to the role of Euclid's Elements in the western mathematics. It is so principal reference for mathematics in Chinese education, a canon both for the construction of mathematics (1st Cent. B.C.-1st Cent. A.D.) and for the teaching/learning of the same in the various historic periods. Among the most notable is the commentary of Liu Hui (263 A.D.) presented in the collection of the Mathematical Canons of the Tang Dynasty (618-907 A.D.). Let us now give a short description of each of the nine chapters of the book.
| Chapter 1: Land Surveying. | This consists of 38 problems on land surveying. It looks first at area problems (the types of shapes for which the area is calculated includes triangles, rectangles, circles, trapeziums), at rules for the addition, subtraction, multiplication and division of fractions. The Euclidean algorithm method for finding the greatest common divisor of two numbers is also presented. In the problem number 32 an accurate approximation is given for π. |
| Chapter 2: Millet and Rice. | This chapter contains 46 problems concerning the exchange rates among twenty different types of grains, beans, and seeds. It possible to find a study of proportion and percentages and an introduction of the rule of three for solving proportion problems. Many of the treated problems apply as simple exercise to give to the reader the practice to work with the calculations with fractions. |
| Chapter 3: Distribution by Proportion. | There are 20 problems which involve proportion (direct proportion, inverse proportion and compound proportion). In particular arithmetic and geometric progressions are used in some of the problems. |
| Chapter 4: Short Width. | 24 problems (the first eleven problems take the name to the chapter). Problems 12 to 18 involve the extraction of square roots, and the remaining problems involve the extraction of cube roots. Notions of limits and infinitesimals appear also in this chapter. |
| Chapter 5: Civil Engineering. | 28 problems on construction of canals, ditches, dykes, etc. it is possible to find volumes of solids such as prisms, pyramids, tetrahedrons, wedges, cylinders and truncated cones Liu Hui, in his commentary, discusses a "method of exhaustion" that he invented to find the correct formula for the volume of a pyramid. |
| Chapter 6: Fair Distribution of Goods. | This chapter contains 28 problems involving ratio and proportion. The problems refer to travelling, taxation, sharing etc. |
| Chapter 7: Excess and Deficit. | 20 problems that report the rule of double false position. |
| Chapter 8: Calculation by Square Tables. | This chapter contains 18 problems which are reduced to solving systems of simultaneous linear equations. However the method given is basically that of solving the system using the augmented matrix of coefficients. The problems involve up to six equations in six unknowns and the only difference with the modern method is that the coefficients are placed in columns rather than rows. The matrix is so reduced to triangular form, using elementary column operations as is done today in the method of Gaussian elimination, and the answer interpreted for the original problem. Negative numbers are used in the matrix and the chapter includes rules to compute with them. |
| Chapter 9: Right angled triangles. | In this final chapter there are 24 problems which are all based on right angled triangles. The first 13 problems are solved using an application of Pythagoras's theorem, which the Chinese knew as the Gougu rule. |

The key concept that organizes the description of the Jiuzhang suanshu is the concept of “class” or “category” (lei) that plays a fundamental role in the commentaries. The elements that we find relevant to understand the specificity of the book and so of the related culture are so: the problems and so the typology of the problematic situation putted on the book and judged relevant for the Chinese culture for the time of the book, the modus operandi described in the book (the “procedure” (shu), the algorithmics in the term sense intended by Chemla (2004, 2007) that are useful to classify, understand and so describe the categories), the calculus instruments, the demonstrations (in the Chinese sense of term), the epistemological values.

The structure of the book is gradually articulated from the simple given of a problem (wen) related to a particular category, to solve it, “generalizing” step by step, trough an analogical reasoning, trough a variable mutation, the proposed situation and defining hence a general solution strictly connected to the proposed contest in a holistic vision (Nisbet, 2001). So, it is through a work on the procedure that is possible to define the situation classes. The solution process is an abductive process where deduction and induction are together in a unique reasoning scheme. The perfection is defined in terms of simplicity and generality trough a global vision of the problematic.

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1 According with the Chinese philosophy in which nothing is clear divided in white and black, neither the colors interpreting the circle Ying e Yang. “The oriental dialectic welcome the possible contradiction inside on a logic reasoning since only trough these the verity is known. (Nisbett, 2003). The fundamental principles that regulate the oriental dialectic are so verifiable on:

a) Principle of mutation: the reality is a process subjected to a constant mutation;
b) Principle of contradiction: since the mutation is constant, the contradiction is also constant;
c) holistic principle: since all the thing varying continually and it is always in contradiction, nothing, in the human life as in nature, is possible to understand independently. All is linked.

All gave them the possibility to tolerate the paradox, it isn’t absolutely absent in the western culture.
As Nisbet declare “The social worlds of East and West today reflect to a substantial degree their origin in Chinese and Greek culture, respectively…the social differences influence cognitive processes…we might expect to find cognitive differences among contemporary peoples that parallel those found in ancient times.

Some of the differences that Nisbet puts in evidence are:
- the relationship between the field and the object, and the perceived relations among events;
- the organization into categories and covering rules, instead of organizing in terms of similarities and relationships (typical for the Chinese culture);
- apparent contradictions, Westerners resolve the situation by deciding which of the two propositions is correct, whereas Easterners are inclined to find some truth in both propositions. Westerners thus emphasize non-contradiction, whereas Easterners value the “Middle Way.”.

Trying to define an universal identity for the features of the East Asian mathematics education with the underlying values in contrast to features and values in the West, Leung in the ICME-9 in Tokyo/Makuhari, Japan, 2000, defined six interesting dichotomies that, according to us, are strictly linked with the other two aspects presented before about the natural written Chinese language and the role of fuzzy logic (an approach of the linguistic type) as interpretive instrument of some problematic situations correlated to the “common sense:

1. Product (content) versus process;
2. Rote learning versus meaningful learning: “Understanding is not a yes or no matter, but a process or a continuum”
3. Studying hard versus enjoying the study “the East Asian view is that learning or studying is necessarily accompanied by hard work, and a deeper level of pleasure or satisfaction is derived only as an end result of the hard work”
4. Extrinsic versus intrinsic motivation
5. Whole class teaching versus individualized learning “Chinese proverb: teaching students in accordance with their aptitude”

Observations on the Chinese written language
For the observations regarding language we are referring to the research works of Chemla (2001), Needman (1981) and Granet (1988).

As Granet declares

“Chinese was able to become a powerful language of civilization and a great literary language without having to worry about either phonetic wealth or graphic convenience, without even trying to create an abstract material of expressions or supplying itself with a syntactic armament. It managed to maintain for its words and sentences a completely concrete emblematic value. It knew how to reserve only for rhythm the care of organizing the expression of thought. As if, above all, it wanted to liberate the spirit from the fear that ideas can become sterile if expressed mechanically and economically, the Chinese language refused
to offer these convenient instruments of specification and apparent coordination which abstract signs and grammatical artifices are. It kept itself obstinately rebellious against formal precisions for the love of the concrete, synthetic adequate expression. Chinese does not seem organized for noting concepts, analyzing ideas or conversationally expressing doctrines. In its completeness, it is constructed for communicating sentimental behaviors, for suggesting conduct, for convincing, for converting.” (Granet, 1988, p.243)

The words are nouns (ming) that refer to “existing things” (wu) in effective reality (shi). As an example we could consider the word that means “old”. It does not exist; in compensation there is a great number of terms which illustrate different aspects of old age, with a full series of subtleties. The Chinese character to express the meaning of “old” is 老 [lǎo] in which 匕 is for 化 huà “change”. 毛 máo means “hair”. "The modern form is an extreme corruption of a seal containing 毛 hair 匕 changing (color): old" (Karlgren, 2002).

The construction of the ideograms are classified in different categories or “meta-rules” of composition. The ideogram presented, in the Chinese writing, is one of the composition rules of the fundamental characters. Needham reports the classification in six classes and he discuss them in this way:

a) Hsiang hsing, lett. «Forms of imagines» (pictograms): tree 木; sun 日; moon 月; mountain 山; horse 马; bird 鸟; crow 哀 (it like 鸟 “bird”, but missing the dot in the head; the eye is invisible because a crow’s eye is black like the feathers);

b) Chih shih, lett. «Indicators of situation» (indirect symbols);

c) Hui i, lett. «Union of ideas» (composition by association or logic composition). 80% of the ideograms are of the associative kind (Needman, 1981). They represent a sort of mental equations as semantic combinations of two or more characters that are composed by association. We could find different examples for this:


Such equations constitute a semiconscious mental foundation for whoever is acquiring familiarity with the language.” (Needham, 1981, pp. 35-36, vol. I).

- 好 [hǎo] good = 女 (nǚ) “woman” and 子 (zǐ) “child”.

The two components combine to represent the meanings “good” and “like”;

- 林 [lín] (森林 sēnlín) forest = tree 木 + tree 木 (plus 木). Two 木 (mù) trees side by side.

- 休 [xiū] stop, rest = 亻 (rén) + 木 (mù) tree. A person stopping to rest under a tree.

d) Chuan chu, lett. «Transferable sense» (symbols that is possible to interpret reciprocally).

e) Hsing sheng, lett. «Language or sound». These characters are defined in a determinate general manner: the radical is associated to a phonetic sign to indicate the category on which we have to find the meaning of the word. So a lot of words with the same sound are written without confusion. (Needham, 1981, p. 38).
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-园 [yuán] garden = 囘 (wéi) “surround”, suggesting a garden fence, and (full form:) 元 yuán phonetic or (simple form:) 元 yuán phonetic;
-袁 yuán or (simple form:) 元 yuán phonetic, and 走 (chuò) “go” (to go far) = “far”

f) Chia chieh, lett. «Loan» (caratteri fonetici in prestito). The formation is much similar to the precedent case but the way to construct the character is different.

Equal definition is reported on all the Chinese grammatical in different volumes on the history of Science in China as in the volumes of Enciclopedia Treccani (1977).

Other two transversal interesting observation could be made regarding the reasoning scheme to the written Chinese language:

  a) the use of the contradiction in a contemporary of the opposites (From the point of view of Fuzzy Logic\(^2\) ) inside the language:

-杯 [bēi] cup: from 木 (mù) “wood” and 不 (bù) “not” phonetic. From the association of these two characters and so from the idea of “opposition/contrast” born the concept of cup: Everybody knows that cups are not made of 木 (wood).
- From 不 (bù) “no” and 口 (kǒu) “mouth” we find the character linked with the idea of not to use the mouth.

  b) the idea of a variable (as thing that varying) and a parametric system inside the composition of many characters. Some simple examples could be:
- 古 (gǔ) = as unitary ideograms “old” composed by “ten” and “mouth” (in reference to the Chinese philosophy That which has passed through 十 ten 口 mouths, i.e. a tradition dating back ten generations) strictly licked to other different characters licked with it by a semantic or phonetic units:
  - 固 = to harden (annoyed and hardened), with the radical 甲: 囉
  - 枯 = to fade (annoyed and done harden) “From 木 (mù) “tree” and 古 gǔ (“old”) phonetic. 古 “old” also it is suggested the meaning, “withered”
  - 故 (gù) = reason, cause (aged, dried him and fixed him) with the radical 66
  - 姑 (gū) = mother-in-law (elderly woman “dried him”) with the radical 66 to the left.
  - 箇 (gē) = solid thing and hardened,
    - (gōng = old men)

Another example in this sense could be with regard to the radical 田 (tián) “field”. We could find other 138 different characters linked with it:
- 里 [lǐ] “village”: From 田 (tián) ‘field’ and 土 (tǔ) ‘earth’. “Village of 25 or 50 families; place of residence; (the length of the side of the said village:) length measure of about 600 meters”. Since the adoption of the metric system, a 里 is exactly 500 meters.
  The word 里 meaning 'inside' in its full form is written with 里 and 衣 (yī) 'clothing',

\(^2\) A set A and the set not-A have in fuzzy logic an intersection which varies from a minimum to a maximum that depends on the possibility of receiving A and not-A and distinguishing A and not-A.
either 裏, split with 衣 on top and the rest below, or 製 with 衣 on the left side. The simple form is just 里 without 衣 (Karlgren, 2002)
- 果 [guǒ] “fruit”: field + tree
- 界 [jiè] “boundary”: From 田 (tián) ‘field’ and 介 jiè ‘introduce’. According to Karlgren, 界 jiè is etymologically the same word as 介 jiè, which originally also meant ‘boundary’.
- 思 [sī] (思想 sīxiǎng) “thought”: The top depicted a brain, now it happens to look like 田 tián ‘field’. The bottom is 心 xīn ‘heart’.
- 备 (F备) [bèi] (准备) “get ready”, “prepare”; 设备 “equipment”: The explanation of 备 is obscure. There have been numerous different forms, including the full form 備 and the simple form 备. Mnemonic for 备: 夂 (zhǐ) walk slowly around a 田 (tián) field, preparing equipment?
- 画 (F画) [huà] “draw”; “picture”: "Draw boundary lines...delineate, draw, paint; drawing, painting; stroke (in writing) -- 莅 to draw 刿 or 一 lines: boundaries of a 田 field" (Karlgren, 2002).

An old form is 亜. In the common full form 亜, the bottom 亜 is reduced to 一. In the simple form 亜 the top 莅 is left out. Compare 亜(亜) zhòu which is similar to 亜(亜) in the full form.

These characteristics of the written language seem to put in evidence an internal research to a use of a common strategy to define “different characters” in which the radicals part assume a role of a parameter (in a mathematical sense of the terms) and vehicle the meaning or the sound of the character. It seems to us a sort of research of a possible fundamental algorithmic to construct different “words” and so to read and to write these in a continuous parallelism, in a continuous relationship, between “serial thought” and “global thought” on each single character.

Ex: Algebra = 代数 = 代 (to represent) + 数 (number) = [人 (men) + 弋 (an arrow that points out: it represents the phonetic part)] + [(弋 (clapping, tapping rhythmically to facilitate in counting) + 坤 (“that is obscure”)]

The ideogram is formed therefore from two meaningful parts that give a new meaning, but at the same time one of the parts also has phonetic value and it communicates the sound.

This observation seems to us to argue how the reasoning pattern inducted from the written natural language brings naturally, unconsciously, the Chinese people to use (in different context) some pre-algebraic reasoning schemes.
Some reflections on “arguing, conjecturing and demonstrating” in Chinese Culture with relation to Occidental Culture.

This paragraph briefly analyses, in a schematic way, some substantial differences founded in the history of the Chinese and Western thought.

In the comparative analysis of science in pre-modern China and the west, Geoffrey E.R. Lloyd (2001, pag. 574) says that, “The aspirations of ancient Greek tradition represented by Euclid, which proposed deducing all mathematics from a single set of indemonstrable but evident axioms were not shared by the Chinese at least until the modern age. In China, as a matter of fact, the goal was not axiomatic-deductive demonstration, but gathering unifying principles from all of mathematics.”

The following table analyses some differences in reasoning patterns in a holistic vision.

<table>
<thead>
<tr>
<th>Occident</th>
<th>Orient</th>
</tr>
</thead>
<tbody>
<tr>
<td>1200 algebra: no formalization</td>
<td>200 B.C. algebra: no formalization</td>
</tr>
<tr>
<td>Paradigm of geometry, Equations</td>
<td>Positional system, matrices (system of the rods)</td>
</tr>
<tr>
<td>Aprioristic formulas that hide the processes, favoring, with the result, determinism</td>
<td>Solving equations by means of algebraic manipulations with the strategies: 1) making equals, 2) making homogeneous, 3) research for fundamental algorithms.</td>
</tr>
<tr>
<td>Reduction ad absurdum in a potential infinite</td>
<td>Existing infinity of operations</td>
</tr>
</tbody>
</table>

What we focus on in this work is the algorithmic aspect and the holistic thought that transpire from the Chinese culture, as it is possible to read through the table and in relation to what we said before about the historical mathematical reference of the Jiuzhang suanshu. According to us and to other research work in the didactics of mathematics, it is in fact one of the main reference for the Chinese mathematical thinking and so for the procedures in argumentation, conjecturation and demonstration. It plays a central role in the Canon of mathematics and also represents a tool to demonstrate. In problem solving, the concept of variable varies and permits, after different steps (algorithmic strategy), to find the unknown value that has to be obtained in the problem. This process for the solution is standard and it is therefore an algorithm. Demonstrating the validity of that reasoning means demonstrating the correctness of the procedure (use of the properties of the operations) in the steps of the algorithm. Thus, the algorithm is a combination of an iteration and of chosen ‘conditionals’. The chosen conditional is a first interesting element of the pattern of reasoning: Iteration; Conditionals (If...then...); Assignment of variables.

The following table attempts to find analogies and differences between the meanings that the algorithm assumes in the two cultures.
<table>
<thead>
<tr>
<th><strong>Intuitive algorithm</strong></th>
<th><strong>From the occidental point of view:</strong></th>
<th><strong>From the oriental point of view</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Formalized algorithm</strong></td>
<td>Algorithm:</td>
<td>A paradigmatic example is the “rule of three”: the rule of three rests on the “quantity of that which one has” and on the pair constructed from the “lü of that which one has” and of the “lü of that which one is looking for” to give rise to the “quantity that one is looking for”.</td>
</tr>
<tr>
<td></td>
<td>1) Effectiveness, actually feasible by an automaton. The automaton must be able to recognize the minimum parts of the description of the algorithm (accepting the language in which the algorithm is written; the well formed sentences are called instructions). 2) Finiteness of expression: finite succession of instructions. Cycles, conditions, jumps. 3) Finiteness of the calculation: in the concept of algorithm there is usually included the condition of termination of the procedure for any situation of initial data within a certain domain. 4) Determinism: at each step of the execution of the procedure one and only one operation must be defined and successively carried out.</td>
<td></td>
</tr>
<tr>
<td><strong>Deterministic algorithm</strong></td>
<td>Condition 1 is inalienable. The others give rise to different types of algorithms. If 4 is missing, the algorithm is called non-deterministic.</td>
<td>Research toward analogies of valid algorithms for classes of homogeneous problems. Reference to the algorithms as true and real models.</td>
</tr>
<tr>
<td><strong>Probabilistic algorithm</strong></td>
<td>Approximate, probabilistic, NP-complete algorithms (if there exists a polynomial algorithm able to confirm whether or not this is effectively the solution of the problem), algorithms that stop after a number of steps which grows exponentially.</td>
<td>Fuzzy algorithms?</td>
</tr>
</tbody>
</table>

In this sense, the algorithm seems as an instrument for demonstrating the precision of an argument (in the Jiuzhang suanshu each argument is concluded with phrases of this type “from here the result”).

Through this cultural aspect another stable reasoning pattern in the Chinese culture is possibly defined: the continuous research to the strategy to “Making homogeneous and making equal”: (from the commentaries of Liu Hui, 263 B.C. (Chemla, 2001, pg. 142))
“Making equal” and “making homogeneous” are strategies of reference to be able to concretize the correctness of the reasoning through an algorithm and it could present, according to us, concrete indications on algebraic manipulations of formulas.

An interesting example of the “Making homogeneous and making equal” is that of the rule of three (from the commentaries of Li Chunfeng, 656 B.C. (Chemla, 2001, pg. 142). This algorithm once again is an operation which “makes equal” and “makes homogeneous” (in the reduction to unity). So, the rule of three, as a fundamental algorithm, is the parallel in western culture of the postulate. The fundamental algorithm can combine itself several times always arriving at a sure argument.

As Liu Hui observes, in this joke of relationships between “serial thought” and “global thought” in reading and understanding a problem, particular attention must be given to the examination of the algorithm on the classes of problems, to be able to highlight its correctness.

**An experience with Italian and Chinese students: theoretical framework**

One of the open problems of the “new” school is to interpret the behavior of students introduced into multicultural classes. In Italy, the issue of "multi-cultural" classes is a phenomenon that, even though rather new, it is in wide expansion: the integration of foreign students in the Italian classes had, in the recent years, an increasing rate and has become no more an exception but, on the contrary, an inevitable reality. «The present situation requires therefore to consider and to reorganize an idea of education in balance with the new needs and resources, in order to strengthen the trend of differences integration, the change and mutual adaptation, an open trial correlated with identities recognition and acceptance and with incorporated knowledge» (Canevaro 1983).

The differences that could be detected in the class activities from this point of view, will turn into sources that enrich the whole class. In these relationships, the teacher has to play the essential role of a “knowledge mediator”. In the specific case of Mathematics, a greater attention was paid in recent years to the problems of the didactics in a multicultural school milieu and these themes were included into several school programs and described in many papers. It becomes evident in this context, that the starting point of any activity facing problems which have arisen out of the presence of cultural differences in the class, is to specify and highlight all moments that characterize cultural models of integration: pupil’s previous knowledge, his motivations, his expectations and abilities, his personal and intellectual characteristics; all that constitutes the necessary prerequisites of every correct pedagogic intervention. (Garcia Hoz-Guerriero-Di Nuovo-Zanniello 2000).

The research activity we propose in this paper belongs to this context. Specifically, it proposes a reflection on teaching/learning methodologies of the concept variable/unknown and therefore on their understanding (augmenting and conjecturing in natural language but not only) in primary school pupils involved in experiments (Chinese and Italian pupils). As we said before, the problem regarding the sense of variable could be connected with some particular aspects of the Chinese culture, for example the structure of the ideograms in the written Chinese language and the logical-
arguementative schemes adopted by the Chinese students in the class. It is also connected (this is the aim of the project in which this research is inserted) with the difficulty to delineate a general framework that can allow to individualize the fundamental steps for the development of the algebraic thought in relationship to the arithmetical and geometrical ones.

In this context, research at national and international level underlines the complex problematic regarding the passage from the arithmetical thought to the algebraic thought and so the birth and the evolution of the sense of the concept of variable for the students. In the phase of transition between arithmetical thought and algebraic thought, they verify then as some epistemological obstacles strictly connected to the passage from a meaningful semantic field, precedent, (the arithmetic) and the syntactic construction of a new language (the algebraic one) can delay the development of the algebraic language and so the algebraic thought. (Spagnolo, 2002)

These experimental analysis allowed us to underline a different behavior of the students in relationship to: the logical structure of the proposed problematic situation, the type of study course attended and the origin country (different culture, different system education, different teaching’s strategies…). Thus, one of the open problems is to interpret the obtained results, in presence of multicultural classes.

The choice to study the Chinese mathematical thought is, as we just said before, due to the fact that the Chinese culture, as regards Natural Language, Philosophy, Logic etc., is the most distant from the western culture; to analyze the reasoning schemes used by the Chinese student in the resolution of a mathematical problem allows us not only to reflect on the differences of argumentation adopted in the two countries in the resolution of a same assignment but, above all, it allows us to reflect on our cultural reference system, the Western one.

- Do the Italian and Chinese students, in the resolution of particular problems, put in evidence different resolution strategies reported to the effect of their origin culture (Natural Language, logical-argumentative schemes, algorithms, etc…)?
- Is it possible to underline these differences analyzing their argumentation and conjecturation on in the passage from the arithmetic thought to the algebraic thought?
- Can the study of such differences help the understanding of the phenomenon of teaching/learning in multicultural situation?

To be able to interpret the comparative study between the Chinese thought and the Italian one in situations of teaching/learning in a multicultural perspective, we are referring to the studies of J. G. Gheverghese (1987) and U. D’Ambrosio (2002).

The principal theoretical reference for the methodology of the study is Brousseau’s theory of the situations (Brousseau 1998); in a multicultural milieu it could result central in the specific phase of socialization of the cognitive styles. To put in evidence the socialization of the cognitive styles (phase of validation of the a-didactic situation) became then the carrying element for the comprehension of the phenomenon.

Having accepted the principle that education comes to be “realized” around the student, considering then the social and physical milieu in which he lives, the Didactics of Mathematics has to build on the different experiences offered by the contemporary
presence in class of different cultures, each one with the own mathematical inheritance and mathematical knowledge, to a cultural exchange and a mutual enrichment.

The topic we deal with allows also a series of transversal theoretical reflections that need to a consider as broad as possible frame of reference that considers not only motivation/emotional side of the didactical activities but also the role of such a didactical methodology, centered on playing and creative activities. (Piaget, 1976, Brousseau, 1998), that “disrupts” that pre-arranged context expected and feared by the pupil in which he carries out mathematics.

Methodology and first results

The problematic situation on witch we are referring on this paper is a particular game, experimented with Chinese children of infancy and elementary school, Sudoku/Magic box opportunely simplified.

The game is the box/matrix shown in the figure aside.

We proposed it in the classroom with other five different images of animals on the cards and a series of rules for the composition/solution of it:

1. each animal cannot be in the same line or column with its enemy
   (we presented the enemy animals);
2. each animal has to appear in the square only once;
3. each student has to insert in the box, all of the nine possible different Animals shown in the image cards;
4. the solution has to be only one.

This is one of the possible games for a first approximation research, conducted in a multicultural milieu with Chinese students and also pupils from other countries, into the relationships between the “serial thought” and the “global thought” in the reading and understanding of the problem. In a first approximation, we could consider, this kind of reasoning connected to the arithmetical and algebraic thinking and their relationship.

We involved in the experiment about 95 children (13 Chinese students) aged 3-10; the age range was chosen to investigate in the broadest possible way the different behaviors and different verbalizations of the pupils in this situation.

The experimentation was divided in two phases:

1. situation/game with children of the infancy School “Ferrara” of Palermo and of the Primary School "Costa G." of Palermo, first two years, to observe through quantitative and qualitative analysis (classroom experiences videotaped), the behaviors enacted by the students and the different playing strategies and the recurrent reasoning of Italian and Chinese students;

2. Semi-structured interviews (videotaped) to two foreign (Chinese) students, inserted in the Sicilian scholastic context at the Elementary school, regarding the same situation/game.

As we previously said the game was chosen and adapted according to a series of critical reflections and research previously carried out within the GRIM on the same topic. To structure the game, we considered some of the particular linguistic aspects that
characterize the structures of Chinese written language and in particular the possibility to interpret an ideogram as union of single elements (local vision, Western vision) or/and unitary character (global vision, Chinese vision), the possibility to find a first approach to the sense of variable inside the written Chinese language and also some of the typical reasoning schemes discussed before and referred to an algorithmic approach to the solution of a mathematical problem including “one problem multiple solutions,” “multiple problems one solution,” and “one problem multiple changes.” (Cai, 1999, 2007)

The selected data, is analyzed both quantitatively, through the analysis of the protocol, and qualitatively with single case studies. For the quantitative analysis we used the software for inferential statistic Chic 3.0 (Classification Hiérarchique Implicative et Cohésitive).

Trough this quantitative analysis of the collected data, shown in the presentation of the article, the proposed game will be examined in relationship to the results underlined previously in other relevant works (realized in other different mathematical contexts) conducted in multicultural milieu with Chinese student of different ages; research work realized within the GRIM.

In this sense, the game of Sudoku/Magic Box seems to confirm, even though it is a first approximation, results previously discussed in other research works: compared to Italian students, Chinese pupils present a different kind of logic in the following items: problem reading data, data organization, “type” of language used to describe the solution and hence different schemes of reasoning in argumentation and conjecturation.

We can therefore consider the situation/game as a first good instrument of investigation of the argumentation, conjecturation and demonstration ability of the involved students. In particular, the collected data relative to the Chinese students, seem to confirm a concrete, pragmatic behavior, already highlighted in the works of Chemla (2001) and Spagnolo (2002); behaviors strictly bound to procedural thinking, to algorithm through which students use each single case (each animal proposed in the game) not only as simple procedural description (each case as a particular problem) but also as a representative of all the possible series, connected through the construction of an algorithm; typical reasoning of construction adopted in the written Chinese language.

This kind of strategy is evident comparing videotapes with the data analysis; it does not appear analyzing strategies adopted by Italian students.

With regard to our research, there are other interesting aspects regarding the way to “read” the box/matrix and discuss it in its “solution”.

The most evident difference between Chinese and autochthonous argumentation is that Chinese students seem to use mainly a pragmatic way of reasoning. During the game it often happens that they try indeed to show the truthfulness of a particular assertion with a sketch or a particular “operation”. The Italian students instead used to justify the adopted strategy, a kind of “local reasoning”, with “theoretical” reference to the scheme of the situation. The chosen “theoretical” references result to be more and more formally rigorous during the game.

From the analysis of the strategies adopted by the students (Italians and Chinese) in the resolution of the assignment, it is also possible to underline the ability of the
Chinese children to read and therefore to interpret the box/matrix proposed in a holistic way, with a global vision. They show therefore attention to the particularity of each single case, each single card image presented in the game, reading the table, in a unitary vision. They underlined immediately, as first step of the game, what was important for the solution of the game, the essential elements of the situation where the data was meaningful for the problem.

Examples of question to Chinese students were:

...We have one “non influence” animals that we can consider only at the end;

...We have animals that can be posed only in one part of the box.

If the Italian children prefer and argued strategies based on attempts and errors, looking for first step, the single relationships among the various image cards (animals) in the game and working on the box for lines or columns and only after in parallel, through lines and columns; the Chinese student, maybe only because, as we just said, of the relationships that is possible to find between this kind of situation/game and some of the linguistic aspect of the Chinese written language, underlines a more uninhibited attitude, working immediately in parallel on lines and columns and reading so the box in an unitary way and justify their behavior with holistic procedures in argumentation and conjecturation.

Other interesting considerations can be driven from the videotaped classroom experience, in particular the interview to the two Chinese students. From this further qualitative analysis, evidently comes out how, in the two cultures (Italian and Chinese), the meaning of the term “To think for cases” is interpreted. Is it a behavior connected to the arithmetical thought, to attempts and errors? It is a scheme for augmenting a solution and conjecture different possible cases? In this sense the proposed activities could allow critical and more careful reflections on the possible correlation between Chinese language, entirely "abstract" and with an “algebraic nature” (in the mathematical meaning) with a complex syntax, Chinese thought and mathematical reasoning schemes (logical-argumentative problematic) adopted by students in class to solve a mathematical problem. In according in fact with idea, the hypothesis, that exists a strong correlation among the Chinese language, at least written language, and the mathematical thought (Spagnolo 2002); this correlation involves also the behavior of students in class when they solve a mathematical problem and to support Hok Wing and Bin (2002, 223-224) who sustain that the Chinese students furnish the tallest performances of the world in assignments that ask the application of mathematical abilities we could consider, as we said previously, this analyzed situation/problem as starting point for future more specific and deepest researches in this context. Particular attention in this sense will be turned, in the future developments of the research to the depended analysis of the algebraic nature of the written Chinese language and the correlation that it could have in the main study of the difficulties showed by the student (western and Chinese) of different grades in the phase of argumentation and conjecturation in the passage from the Arithmetical thought to the Algebraic one, from “To think for cases” in the arithmetical acception to the final formalization; problems already well documented and discussed in literature when looking at Western research.
References


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Different procedures in argumentation and conjecturation in primary school


