A study of the effectiveness of a Dynamic Geometry Program to support the learning of geometrical concepts of 2D shapes

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Abstract
The study described in this paper aims to investigate 12-13 year old students’ geometrical reasoning in a Dynamic Geometry environment in order to answer the question: “In what ways do the affordances of Dynamic Geometry Software act to mediate the learning of geometrical concepts of 2 dimensional shapes?” This paper reviews literature describing theories on how humans conceptualise geometric figures, and how this can sometimes lead to misconceptions. Dynamic Geometry Software may hold some of the answers to the problems students have with conceptualising shapes in geometry, and support the development of geometrical reasoning and the correct use of mathematical language. The importance of designing a task which is seen by the students as having a purpose and how the drag mode of Dynamic Geometry can support this is also mentioned.

A pilot study has already been undertaken with pairs of students with the objective of gaining insight into the students’ reasoning about the subject matter and how this develops as they work with the computer on the tasks using the drag mode and Measures menu of the software. As such the study takes an interpretivist stance because it is concerned with the thought processes of human beings which are complex and best analysed using qualitative methods. It is also located in a socio-cultural paradigm because the experimental situation is very much a social situation between two students working with the computer (a tool) and one researcher acting as teacher.

Introduction
The context of the study is the mathematics classroom in England where Geometry has a low profile compared to Number and Algebra. In national tests the questions on the properties of 2D shapes require a low level of reasoning and yet are poorly answered by most students (QCDA, 2004). In this study I plan to use a task set in a Dynamic Geometry Software (DGS) environment to provide an engaging opportunity for students to learn and operate with the properties of 2D shapes and in so doing I hope to gain an insight into how the software can support the development of geometric thinking.

How does the human mind conceptualise geometrical figures?
At first glance geometry appears to be a practical area of mathematics but it is actually very abstract. Consider a 2D shape such as a kite. Researchers have suggested that problems in geometry arise because students find it hard to appreciate the difference between the actual figure (of a kite) on paper and the theoretical object that it represents (Battista 2007). In fact Love (1995) suggests that geometry is the only area of mathematics where the physical geometrical image is both the actual object and its representation.

Fischbein (1993) described students’ mental images of 2D shapes in three ways. Firstly there is the image of a figure based on the material representation of it such as a diagram or a plastic tile. The second way that humans conceptualise shapes is by their definitions. A definition is a minimum set of properties which describe the shape. In the example of a kite a common definition is that of two pairs of adjacent equal sides. Finally Fischbein described the figural concept which is the mental reality students manipulate when they reason or solve problems in geometry, the ideal perfect shape imagined in the mind and that a diagram merely represents. When students operate on a geometrical figure they are operating on its figural concept, the idealised shape, ignoring any other qualities its diagrammatic representation might have eg line thickness, colour and even orientation. By
operating on a geometrical figure students are fusing the conceptual properties (definition) of the figure with its figural concept which is a highly intellectual activity using the axioms of geometry. Students often have difficulty working with the figural concept because they are directly aware of the image which is based upon their experience of material representation: drawings, tiles, etc. Fischbein argues that the ability to work with figural concepts needs to be specifically developed. Another way of analysing how students conceptualise 2D shapes is described by Duval, (1995) which he called ‘cognitive apprehension.’ It has four aspects.

**Perceptual apprehension** is made up of the understanding of the properties of a figure with its visual aspects. Difficulties arise when students perceive that specific aspects of the drawing of the figure, such as the way the figure is orientated, are the properties of the figure.

**Sequential apprehension** is concerned with the understanding of the process used to construct the figure and this process is dependent on the tools used. Gomes and Vergnaud (2004) showed that students who constructed geometrical objects on paper using pencil and compasses used a different set of geometric properties than when they constructed them using DGS. Gomes and Vergnaud concluded that the learning and conceptualisation of geometry is different when using ruler and compasses and when using DGS.

**Discursive apprehension** relates to the way that students think about and describe the figure. **Operative apprehension** describes the way students may gain insight into a figure by operating on it in some way, for example, dividing the figure into smaller figures, performing a transformation on it, etc.

Duval goes on to say that the different types of apprehension overlap, that operative apprehension always overlaps with one of the others and that the different kinds of cognitive apprehension need to be taught separately to students. Finally he suggests that while it is obvious that computer geometry supports sequential apprehension it may also support the development of operative apprehension.

**Dynamic Geometry Software (DGS) may address the problems students have with conceptions of geometrical figures**

I believe that DGS may act as the mediator for the figural concept and as such it has potential for supporting students’ learning of geometrical concepts.

An important affordance of DGS is the dragging mode whereby objects on the screen can be manipulated whilst keeping all constructed properties constant. This helps students to make the link between geometrical concepts and geometric constructions and indicates that DGS has accomplished the link between geometry and the experimental field of geometric constructions (Laborde, 1993). The dragging mode thus acts to mediate the concept of geometrical relationships and helps students to see the relationships between objects (points, lines, etc) rather than focusing on the objects themselves (Holzl et al, 1994).

Jones (2000) also notes that DGS has introduced a way to validate a construction through the dragging feature. To pass the ‘drag test’ a figure has to be constructed in such a way that is consistent with geometrical theory. For example an isosceles triangle will only remain so under dragging if its properties have been constructed into the figure. Thus the drag test can provide the motivation for students to learn about geometrical principles. As Jones states; the dynamic nature of the software influences how students reason about geometrical objects.

However an important concept is the hierarchy of functional dependences which relates to the way that constructed objects depend on the original objects used in the construction (Holzl et al, 1994). If the students fail to understand the hierarchy they may fail to correctly construct geometrical figures and this may have an effect on their ability to use DGS to learn about geometrical concepts. Olive (2000) states that “Dynamic geometry environments can (and should) transform the teaching and learning of mathematics” (p. 228) and that Dynamic Geometry Software turns mathematics into a practical science where students can observe, record, manipulate, predict, conjecture, test and theorise.
Students can use the drag mode and the measure facility to move between practical and theoretical geometry

The Measure menu is another affordance of DGS which allows students to measure objects such as lengths and angles. As the figure is dragged the measures continually update. Olivero and Robutti (2007) conducted a study of 15-16 year old students in Italy and observed that they used the drag mode and the Measuring facility to move between the spatio-graphical field, the experimental or practical side of geometry where they felt most comfortable, and the theoretical field, the area of geometrical theory, concepts and understanding where they needed to develop more sophisticated levels of geometrical reasoning.

Hollebrands (2007) conducted a study of 15 and 16 year old students in the USA and observed how they used the drag mode to investigate geometrical transformations of figures on the screen. She noted that her students used the Measure facility with the drag mode to explore relationships, create and verify conjectures and check the correctness of constructions. Students can do this in a reactive way by dragging in a fairly random fashion in order to see what happens or in a proactive way when then they are able to predict the outcome of their actions before carrying them out. Encouraging students to use strategies that are more proactive may be achieved by asking students to explain and justify what happens on the computer screen in terms of geometrical properties. However Hollebrands found that the dragging mode is only useful for learning if students know and understand how to use it to investigate the variance and invariance in geometrical objects constructed in DGS.

Dynamic Geometry Software (DGS) as a mediator for the figural concept

Using DGS can provide students a means to understand the properties of geometrical figures. In paper geometry, theoretical objects (figural concepts in Fischbein’s terms) are mediated by their material representations on paper (Laborde, 1993). These representations are imperfect, for example, the lines have width in the drawing. Mathematicians ignore the imperfections and work on the drawings as if they were the idealised geometrical object. Laborde explains that the introduction of DGS enables us to redefine the distinction between the theoretical object and its material representation. There is now a figure on the screen and this figure is a new kind of mediator for the theoretical object. It is different from a paper drawing in that it is dynamic (can be dragged on the screen) and its behaviour when dragged is determined by the method used to construct it, that is the geometrical properties designed into its construction.

Mariotti (1995) extends this point by claiming that drawings act as mediators between concrete and theoretical objects. Screen images represent the external version of the figural concept. To construct an object in a dynamic geometry environment the conceptual and figural aspects must be made explicit in the construction process. In this way working in a dynamic geometry environment is useful to develop the correct interaction between the figural and conceptual aspects of geometrical reasoning. The internal logic of the geometrical figure becomes apparent when it is dragged since the geometrical relationships it has been defined by remain constant under dragging.

Hollebrands (2007) argues that computer environments can help to mediate students’ understandings of geometrical transformations when they construct objects in DGS and operate on these objects. However the students then need to reason about the abstract geometrical objects whilst they are interacting with their representations on the computer screen. Hollebrands goes on to say that in order to construct geometrical objects which maintain the expected properties the student needs knowledge of the mathematics and also of the tool (DGS).

Explanation and justification in a DGS environment

Working with computer software can mediate learning through the language and notational system that is designed into the program (Hollebrands, 2007). When the teacher and student interact while using DGS they both adopt the language of the software, a language they then use to communicate with each other. They can also communicate mathematical ideas through discussing the visual images on the screen.
Jones (2000) in a study with thirteen year old students in England, also noted how the use of students’ mathematical language developed while using DGS; from simply describing what was happening on the computer screen to being able to give explanations relating to the mathematical content using more mathematical terminology.

De Villiers (2007) argues for the importance of explanation and justification. He commented that rather than requiring school students to use formal proof in geometry it is more useful to ask them why they think a particular result is true and to get them to explain why they think this is. He argues that proof can be introduced as a means to explain, to make discoveries and to verify conjectures.

**Designing tasks that enhance students’ learning in a computer environment**

The computer offers ways of working that helps students to access approaches and solutions which would not be available to them using pencil and paper (Hoyles and Noss, 1992). However students will not necessarily appreciate the intended mathematical ideas just because they are interacting in a particular computer environment. Tasks need to be designed with the pedagogical principles built into them.

Ainley, Pratt and Hansen (2006) and De Villiers (1994) argue that is necessary to address both the mathematical learning and the motivation issues when designing a task. The task needs to have a purpose and it is this purpose which helps the students to appreciate the utility of the mathematics they need to learn. The idea of creating drag proof figures in DGS may provide a meaningful task for students working in a Dynamic Geometry environment as Healy et al (1994) described when they designed a task using DGS as a vehicle for introducing geometrical constructions. The 13 year old students were asked to create a picture on the screen which was resistant to dragging. The students found this concept to be a meaningful idea and it gave an acceptable form of validation for constructions.

**The research experiment**

The research experiment is based around the design of tasks to support geometrical reasoning in the properties of 2D shapes. The student subjects are 13 year olds working in pairs with one computer which has been loaded with the Geometers Sketchpad (GSP) (Jackiw, 2001), in which files have been created for the tasks. The students take turn to control the computer mouse and are encouraged to work together and discuss the activity.

In the first session the students are given files containing two perpendicular bars. They are told to join the ends of the lines to make a shape. They then drag the bars to generate different triangles and quadrilaterals. They convince themselves and the researcher that they have a particular shape by measuring lines and angles. When they have made a particular shape they are asked to describe the orientation of the bars with respect to each other.

The figure below shows a screen shot taken from one of the screen recordings during the first task. The students have dragged the bars to make a shape very close to a rhombus.
For the second session the students are asked to construct certain shapes starting with a blank screen. They are encouraged to remember what they noticed about the orientation of the bars in the shapes from the first session as this may help in constructing them. In preparation for this session a number of help cards have been designed which give instructions on how to make various constructions such as the perpendicular to a line.

The pilot study highlighted some issues which I plan to address in the larger project:

- The use by the students of the drag mode and the measure facility and how they are used together to support their learning of the properties of 2D shapes
- The development of the mathematical language used by the students to mirror the language used by the software and the help cards
- The students’ internalisation of the need to construct ‘drag-proof’ shapes and whether this will lead to a development of students’ reasoning about the properties of shapes.

**Data collection and analysis**

The data will be in the form of an audio recording of the dialogue during the sessions and a Camtasia recording of the computer screen activity (a version of Image Capture Software which records activity on the computer screen). The activity on the computer screen and the dialogue will be coded in order to ascertain themes which may indicate how the students are creating meaning from their activities.

Some questions I plan to ask in order to encourage the students to focus on particular aspects of the subject matter are:

- What shape have you made?
- If that shape is a kite (or square, rhombus, etc), then what must be true?
- What can you tell me about how the bars are positioned with each other?
- Is that shape drag proof? Why do you think it is drag proof?
References
De Villiers, M., 2007, Proof in Dynamic geometry: More than Verification. *9th International Conference in Mathematics Education in the Global Community*